1. Precision Solution for ψ_s

2. Solution for V_I with DR Approximation Form

3. DR Approximation – ψ_{sa} Reference

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5. DR Approximation – ψ_{sS} Reference – Level 3 Drain Current Solution

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9. Source Reference From Taylor Series Version – Level 3

10. Pinch-Off Reference From Taylor Series Version – Strong Inversion

1. Precision Solution for Surface Potential ψ_s

Sum of Gate Voltage Terms

$$V_G = V_{FB} + \psi_s + V_C \qquad 1$$

Gate Voltage Term for All Charge Under the Oxide

$$V_C = \gamma\sqrt{\psi_s + V_{th}\exp\left(\frac{\psi_s - PHI}{V_{th}}\right)} \qquad 2$$

Gate Voltage for Channel

$$V_I = \gamma\sqrt{\psi_s + V_{th}\exp\left(\frac{\psi_s - PHI}{V_{th}}\right)} - V_B \qquad 3$$

Gate Voltage for Ion Depletion-Region Charge

$$V_B = \gamma\sqrt{\psi_s} \qquad 4$$

Iteration Solution

$$V_{Gp} = V_G - V_{FB} \qquad 5$$

$$H = \exp\left(\frac{-PHI - V_{ch}}{V_{th}}\right) \qquad 6$$

$$f = \left(V_{Gp} - \psi_s\right)^2 - \gamma^2\psi_s - V_{th}\gamma^2 H\exp\left(\frac{\psi_s}{V_{th}}\right) \qquad 7$$

$$f_d = -2V_{Gp} + 2\psi_s - \gamma^2\left[1 + H\exp\left(\frac{\psi_s}{V_{th}}\right)\right] \qquad 8$$

$$\psi_s = \psi_s - \frac{f}{f_d} \qquad 9$$

2. Solution for V_I with DR Approximation Form (Eq. 27.)

$$V_{Ba} = \gamma\sqrt{\psi_{sa}} \qquad 10$$

$$\alpha_a = 1 + \frac{\gamma}{2\sqrt{\psi_{sa}}} \qquad 11$$

$$\psi_s = \psi_{sa} - \frac{V_I}{\alpha_a} \qquad 12$$

$$C = V_{th}\gamma^2 \exp\left(\frac{\psi_{sa} - PHI - V_{ch}}{V_{th}}\right) \qquad 13$$

$$V_I = \sqrt{V_B{}^2 + C\exp\left(\frac{-V_I}{\alpha_a V_{th}}\right)} - V_B \qquad 14$$

$$V_I{}^2 + 2V_B V_I = C\exp\left(\frac{-V_I}{\alpha_a V_{th}}\right) \qquad 15$$

$$A_\alpha = \frac{\alpha_a - 1}{\alpha_a} \qquad 16$$

$$V_B = V_{Ba} - A_\alpha V_I \qquad 17$$

$$A = \frac{\alpha_a - 2}{\alpha_a} \qquad 18$$

$$\frac{A}{C}V_I{}^2 + 2\frac{V_{Ba}}{C}V_I = \exp\left(\frac{-V_I}{\alpha_a V_{th}}\right) \qquad 19$$

$$V_I = -\alpha_a V_{th} \ln\left(\frac{A}{C}V_I{}^2 + 2\frac{V_{Ba}}{C}V_I\right) \qquad 20$$

$$f = \alpha_a V_{th} \ln\left(\frac{A}{C} V_I^{\,2} + 2\frac{V_{Ba}}{C} V_I\right) + V_I \qquad 21$$

$$f_d = \alpha_a V_{th} \frac{2V_{Ba} + 2AV_I}{2V_{Ba} + AV_I^{\,2}} + 1 \qquad 22$$

$$V_I = V_I - \frac{f}{f_d} \qquad 23$$

Omit $V_I^{\,2}$ Term

$$f = \alpha_a V_{th} \ln\left(2\frac{V_{Ba}}{C} V_I\right) + V_I \qquad 24$$

$$f_d = \alpha_a \frac{V_{th}}{V_I} + 1 \qquad 25$$

$$V_I = V_I - \frac{f}{f_d} \qquad 26$$

$$i_{Ddrift} = \frac{1}{2\alpha_a}\left(V_{IS}^{\,2} - V_{ID}^{\,2}\right) \qquad V_{ch} = V_S \text{ and } V_D \text{ in } C \text{ Eq. 13}$$

3. DR Approximation – ψ_{sa} Reference

$$V_B = \gamma\sqrt{\psi_s} \approx \gamma\sqrt{\psi_{sa}} + \frac{\gamma}{2\sqrt{\psi_{sa}}}\left(\psi_s - \psi_{sa}\right) \quad (27)$$

$$V_I = V_G - V_{FB} - \psi_s - \left(\gamma\sqrt{\psi_{sa}} + \frac{\gamma}{2\sqrt{\psi_{sa}}}\left(\psi_s - \psi_{sa}\right)\right) \quad (28)$$

$$V_{Ta} = V_{FB} + \psi_{sa} + \gamma\sqrt{\psi_{sa}} \quad (29)$$

$$V_G - V_{Ta} = 0 \quad (30)$$

$$V_I = \alpha_a\left(\psi_{sa} - \psi_s\right) \quad (31)$$

$$\psi_s = \psi_{sa} - \frac{V_I}{\alpha_a} \quad (32)$$

$$\alpha_a = 1 + \frac{\gamma}{2\sqrt{\psi_{sa}}} \quad (33)$$

$$\frac{\partial V_I}{\partial \psi_s} = -\alpha_a \quad (34)$$

4. Drain Current – Strong Inversion

$$V_I = \alpha_a \left(\psi_{sa} - \psi_s\right) \qquad \text{Eq. 31}$$

$$i_{Ddrift} = \frac{\alpha_a}{2}\left(\left(\psi_{sa} - \psi_{sS}\right)^2 - \left(\psi_{sa} - \psi_{sD}\right)^2\right) \qquad 35$$

$$\psi_s = \psi_o + V_{ch} \qquad 36$$

$$\psi_o = PHI + nV_{th} \qquad 37$$

$$i_{Ddrift} = \frac{\alpha_a}{2}\left(\left(\psi_{sa} - \psi_o - V_S\right)^2 - \left(\psi_{sa} - \psi_o - V_D\right)^2\right) \qquad 38$$

$$V_P = \psi_{sa} - \psi_o \qquad 39$$

$$i_{Ddrift} = \frac{\alpha_a}{2}\left(\left(V_P - V_S\right)^2 - \left(V_P - V_D\right)^2\right) \qquad 40$$

5. DR Approximation ψ_{sS} Reference – Level 3 Drain Current Solution

$$\alpha_s = 1 + \frac{\gamma}{2\sqrt{\psi_{sS}}} \qquad 41$$

$$V_T = V_{FB} + \psi_{sS} + \gamma\sqrt{\psi_{sS}} \qquad 42$$

$$V_P = \frac{V_G - V_T}{\alpha_s} \qquad 43$$

$$V_{IS} = \alpha_s V_{Ps} \qquad 44$$

$$V_{ID} = \alpha_s \left[V_{Ps} - \left(\psi_{sD} - \psi_{sD} \right) \right] \qquad 45$$

$$i_{Ddrift} = \frac{1}{2\alpha_s}\left(V_{IS}{}^2 - V_{ID}{}^2 \right) \qquad 46$$

$$i_{Ddrift} = \frac{\alpha_s}{2}\left[V_P{}^2 - \left(V_P - \left(\psi_{sD} - \psi_{sS} \right) \right)^2 \right] \qquad 47$$

$$i_{Ddrift} = \alpha_s \left[V_P \left(\psi_{sD} - \psi_{sS} \right) - \frac{\left(\psi_{sD} - \psi_{sS} \right)^2}{2} \right] \qquad 48$$

Strong Inversion – Level 3

$$V_{GS} = V_G - V_D \qquad 49$$

$$V_{DS} = V_D - V_S \qquad 50$$

$$V_{Ts} = V_{FB} + PHI + \gamma\sqrt{PHI + V_S} \qquad 51$$

$$\alpha_s = 1 + \frac{\gamma}{2\sqrt{PHI + V_S}} \qquad 52$$

$$V_{Ps} = \frac{V_G - V_{Ts}}{\alpha_s} \qquad 53$$

$$i_{Ddrift} = \alpha_s\left(V_P V_{DS} - \frac{V_{DS}{}^2}{2}\right) \qquad 54$$

$$i_{Ddrift} = \left(V_G - V_{Ts}\right)V_{DS} - \alpha_s \frac{V_{DS}{}^2}{2} \qquad 55$$

Saturation

$$i_{Ddrift} = \frac{\alpha_s}{2} V_P{}^2 \qquad 56$$

6. Drain Current Solutions – Case C – Precision

$$I_{Ddif} = -\beta V_{th} \int_{V_{IS}}^{V_{ID}} dV_I = \beta V_{th} \left(V_{IS} - V_{ID}\right) \quad 57$$

$$I_{Ddrift} = \beta \int_{\psi_{sS}}^{\psi_{sD}} V_I d\psi_s \quad 58$$

$$V_I = V_G - V_{FB} - \psi_s - \gamma\sqrt{\psi_s} \quad 59$$

$$I_D = I_{Dif} + I_{Drift} \quad 60$$

$$I_{Ddrift} = \beta\left[\left(V_G - V_{FB}\right)\left(\psi_{sD} - \psi_{sS}\right) - \frac{1}{2}\left(\psi_{sD}{}^2 - \psi_{sS}{}^2\right) - \frac{2}{3}\gamma\left(\psi_{sD}{}^{\frac{3}{2}} - \psi_{sS}{}^{\frac{3}{2}}\right)\right]$$

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$$I_{Ddif} = \beta V_{th}\left[\left(\psi_{sD} - \psi_{sS}\right) + \gamma\left(\psi_{sD}{}^{\frac{1}{2}} - \psi_{sS}{}^{\frac{1}{2}}\right)\right] \quad 62$$

7. Case A - DR Approximation

$$I_{Ddrift} = \beta \int_{\psi_{sS}}^{\psi_{sD}} V_I d\psi_s \quad 63$$

$$\frac{\partial V_I}{\partial \psi_s} = -\alpha_a \quad 64$$

$$I_{Ddrift} = -\frac{\beta}{\alpha_a} \int_{V_{IS}}^{V_{ID}} V_I dV_I = \frac{\beta}{2\alpha_a}\left[V_{IS}^2 - V_{ID}^2\right] \quad 65$$

$$I_{Ddif} = -\beta V_{th} \int_{V_{IS}}^{V_{ID}} dV_I = \beta V_{th}\left(V_{IS} - V_{ID}\right) \quad 66$$

8. Taylor Series Approximation to 3/2 Term in Eq. 61

$$f = \gamma \frac{2}{3} \psi_s^{3/2} \quad (67)$$

$$f_o = \gamma \frac{2}{3} \psi_{sa}^{3/2}$$

$$f_d = \gamma \sqrt{\psi_{sa}}$$

$$f_{d2} = \frac{\gamma}{2\sqrt{\psi_{sa}}}$$

$$\alpha = 1 + \frac{\gamma}{2\sqrt{\psi_{sa}}}$$

$$f_S = f_o + f_d(\psi_{sS} - \psi_{sa}) + \frac{f_{d2}}{2}(\psi_{sS} - \psi_{sa})^2 \quad (68)$$

$$f_D = f_o + f_d(\psi_{sD} - \psi_{sa}) + \frac{f_{d2}}{2}(\psi_{sD} - \psi_{sa})^2$$

$$f_3 = \gamma\left[\frac{2}{3}\psi_{sD}^{3/2} - \frac{2}{3}\psi_{sS}^{3/2}\right]$$

$$f_3 = \left[\gamma\sqrt{\psi_{sa}}(\psi_{sD} - \psi_{sS})\right] + \frac{\alpha - 1}{2}\left[(\psi_{sD} - \psi_{sa})^2 - (\psi_{sS} - \psi_{sa})^2\right]$$

$$i_{Ddrift} = (V_G - V_{FB})(\psi_{sD} - \psi_{sS}) - \frac{1}{2}\left(\psi_{sD}^2 - \psi_{sS}^2\right) - f_3 \quad (69)$$

$$i_{Ddrift} = (V_G - V_T)(\psi_{sD} - \psi_{sS}) - \frac{\alpha}{2}\left[(\psi_{sD} - \psi_{sa})^2 - (\psi_{sS} - \psi_{sa})^2\right] \quad (70)$$

$$V_T = V_{FB} + \psi_{sa} + \gamma\sqrt{\psi_{sa}} \quad (71)$$

9. Source Reference – Level 3

$$\psi_{sa} = \psi_{sS}$$ 72

$$\alpha = 1 + \frac{\gamma}{2\sqrt{\psi_{sS}}}$$ 73

$$V_T = V_{FB} + \psi_{sS} + \gamma\sqrt{\psi_{sS}}$$ 74

$$i_{Ddrift} = (V_G - V_T)(\psi_{sD} - \psi_{sS}) - \frac{\alpha}{2}(\psi_{sD} - \psi_{sS})^2$$ 75

Strong Inversion – Level 3

$$\alpha = 1 + \frac{\gamma}{2\sqrt{PHI + V_S}}$$ 76

$$V_{DS} = (\psi_{sD} - \psi_{sS})$$ 77

$$i_{Ddrift} = (V_{GS} - V_T)V_{DS} - \frac{\alpha}{2}V_{DS}{}^2$$ 78

$$V_{To} = V_{FB} + PHI + \gamma\sqrt{PHI}$$ 79

$$V_T = V_{To} + \gamma\left(\sqrt{PHI + V_S} - \sqrt{PHI}\right)$$ 80

$$V_{DSsat} = \frac{V_G - V_T}{\alpha}$$ 81

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10. Pinch-Off Reference – Strong Inversion

$$i_{Ddrift} = \frac{\alpha}{2}\left[\left(\psi_{sS} - \psi_{sa}\right)^2 - \left(\psi_{sD} - \psi_{sa}\right)^2\right] \quad 83$$

With Depletion-Region Approximation

$$V_I = \alpha\left(\psi_{sa} - \psi_s\right) \quad 84$$

$$i_{Ddrift} = \frac{1}{2\alpha}\left(V_{IS}{}^2 - V_{ID}{}^2\right) \quad 85$$

Strong Inversion (EKV)

$$\psi_o = PHI + nV_{th} \quad 86$$

$$V_P = \psi_{sa} - \psi_o \quad 87$$

$$\psi_{sD} = V_D + \psi_o \quad 88$$

$$\psi_{sS} = V_S + \psi_o \quad 89$$

$$i_{Ddrift} = \frac{\alpha}{2}\left[\left(V_S - V_P\right)^2 - \left(V_D - V_P\right)^2\right] \quad 90$$